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AEROTHERMOOPTICS

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Abstract—On the example of thermal gas lenses (TGL) a survey is presented of principles and methods of aerothermooptics. The relationship is established between optical and thermohydrodynamic parameters of TGL with regard for free convection. Velocity and temperature fields are determined in TGL, including those rotating around their axes. The analytical expressions are obtained for aberration coefficients in the eikonal expansion up to the fourth-order wave aberrations. Rotation of TGL is known to be an effective means of free convection suppression and symmetrization of optical characteristics. Perspectives are discussed of the further development of aerothermooptics.

NOMENCLATURE

$n(\mathbf{r})$,	refractive index;
ds,	elements of beam;
r,	point coordinate on beam;
$\mathbf{S}$,	unit vector parallel to beam;
∇,	gradient operator;
ρ,	density;
$\mathbf{v}$,	velocity;
g,	gravity;
μ,	shear viscosity;
P,	pressure;
∇^2,	Laplace operator;
C_P,	heat capacity at constant pressure;
T,	temperature;
α,	molecule polarizability;
K,	Boltzmann constant;
θ,	dimensionless temperature;
δ,	beam aberration;
$V(\mathbf{r},\mathbf{r}')$,	eikonal;
P_0, P_1',	point on the plane of object and intersection of beam with Gauss sphere, respectively;
C_i,	aberration coefficients.

AEROTHERMOOPTICS, a perspective scientific trend, which in recent years Academician A. V. Luikov took a lively interest in, is synthesized at the junction of electrodynamics, aerodynamics and heat and mass transfer and deals with interaction of electromagnetic waves (mainly of optical ranges) with media involving heat- and mass-transfer processes. In the introduction to his book [1] Luikov pointed out as one of the principal problems of aerothermooptics "obtaining media with required optical properties using thermal, aerodynamic and concentration means as well as determination of heat- and mass-transfer characteristics by optical methods". Though the formulated problems may seem to be referred to electrodynamics of continua, their peculiarities, however, make it possible to single them out into a special branch with equal reasons as optics is branched off electrodynamics. Indeed, although complete information on propagation of any frequency waves is contained in the Maxwell equations, specific methods are developed for every range which have become independent scientific branches.

In the present paper the subject and methods of aerothermooptics will be demonstrated on the example of one particular problem of creating and describing thermal gas lenses (TGL).

The first TGL was proposed by Berriman [2]. Later on many gas lenses (GL) of different designs and operating principles were suggested (see [3] and the literature cited there).

TGL were proposed as a means to compensate for natural diffraction divergence of a laser beam as an alternative of usual glass lenses which due to a number of reasons (great losses for absorption, reflection and dispersion) turned out to be unsuitable for light guides used in optical communications.

A typical use of TGL is for creation of a hydrodynamic flow with the prescribed characteristics in a channel of certain geometry with the required thermal conditions to be kept on its walls. In this case values of thermal, hydrodynamic, geometrical and optical characteristics of TGL appeared to be interrelated. Establishment of the relation is just the problem of TGL theory.

Keeping in mind that optical waves propagation is satisfactorily described within the framework of geometrical optics [4], further we shall make use of the trajectory equation

$$\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) = \nabla n. \qquad (1)$$

Here ds is the trajectory element; $\mathbf{r}$ is the coordinate of a point on a ray; $\mathbf{S} = d\mathbf{r}/ds$, is the unit vector tangential to the ray; $n(\mathbf{r})$ is the coordinate-dependent refractive index of the medium in which light propagates. Developing the required distribution $n(\mathbf{r})$ in the medium we may control propagation of light beams.

Consider an incompressible gas flow in a circular cylindrical tube with heated walls in a gravitational field. The constitutive equations describing such a flow (only a steady state will here be considered) comprises the continuity equation

$$\nabla.(\rho\mathbf{v}) = 0 \qquad (2)$$

the equation of motion

$$\rho\mathbf{v}.\nabla\mathbf{v}+\nabla P-\rho\mathbf{g}-\mu\nabla^2\mathbf{v} = 0 \qquad (3)$$

and the equation of energy

$$\rho \mathbf{v}.\nabla(C_P T) = \lambda \nabla^2 T + \mathbf{v}.\nabla P + \Phi \quad (4)$$

wherein $\mathbf{g}$ being the gravity, Φ being the dissipative function. The system of equations (1)–(4), generally speaking, describes ray propagation in the medium considered. The necessary relation between solution of system (2)–(4) and $n(\mathbf{r})$ in (1) may be found on the basis of the Clausius–Mosotti equation [5]

$$\frac{n^2-1}{n^2+2} = \frac{1}{3} N\alpha \quad (5)$$

and equations for an ideal-gas state

$$P = NKT \quad (6)$$

where N is a number of molecules per unit volume, α is the polarizability of an individual molecule, P is the pressure, T is the absolute temperature (°K); K is the Boltzmann constant.

Thus, solution of equations (1)–(6) provides a description of light ray propagation in a gas medium, motion of which is governed by equations (2)–(4).

In contrast to the traditional approach to the problems of hydrodynamics and heat transfer when the main efforts of investigators are directed towards determination of velocity and temperature gradients on the channel walls and dimensionless relations for prediction of friction and heat transfer [6], peculiarities of the local flow structures and temperature fields within the whole channel cross-section to be revealed.

The solution of system (2)–(4) is connected with an account of the free convection effect on forced one that may be characterized by the natural parameter $\xi = Gr/Re^2$ where Gr and Re are Grashof and Reynolds numbers. For a region of values of interest, ξ is less than unity that allows for the small parameter method to be used for the solution of (2)–(4). Using the Bousinesque approximation, adhesion condition for velocities and the second kind boundary conditions for heat transfer, a solution for dimensionless temperature (accurate to ξ^2) may be obtained in the form [7]

$$\theta = r^2 - \frac{r^4}{4} - \frac{3}{4} + \frac{Gr}{Re^2} \sum_{n=1}^{6} C_n r^{2n-1} \cos\varphi + \left(\frac{Gr}{Re^2}\right)^2 \sum_{n=1}^{10} (\alpha_n + \beta_n \cos 2\varphi) r^{2n-2}. \quad (7)$$

Explicit expressions are available for coefficients $C_n(Re, Pe)$, $\alpha_n(Re, Pe)$ and $\beta_n(Re, Pe)$ entering into this solution [7].

The solution differs from expressions for w and θ in [8–10] by additional summands accounting for a change of an axial pressure gradient due to free convection and may be considerable in the range of small Pe.

With available solution (7) for a temperature field it might be possible to determine through (5) a refractive index field in a lens and the ray trajectory by solving equation (1). However, the essentially non-linear behaviour of $\theta(r, \varphi)$ does not permit in a general case analytical expressions for the trajectory to be obtained. Due to this reason we take advantage of a less strict method but which makes it possible to get more obvious results.

Beam aberration equal to difference between pulses of real ($\mathbf{r}_1$) and paraxial ($\mathbf{r}_1^*$) images is determined as

$$\delta = -\frac{D}{n_0} \nabla_{r_1'} V'(\mathbf{r}_0, \mathbf{r}_1') \quad (8)$$

(primed V' means that the terms corresponding to the paraxial image are eliminated from it, and projection of all the vectors, but $\mathbf{r}$, in (9) on axis z are equal to zero) where $\mathbf{r}_0$ and $\mathbf{r}_1$ are the coordinates of ray inlet and outlet, respectively; n_0 is the refractive index outside the lens; D is the distance between the outlet eye and the image plane; V is the point eikonal defined as

$$V(\mathbf{r}_0, \mathbf{r}_1') = \int_{P_0}^{P_1'} n(r)\,\mathrm{d}s. \quad (9)$$

Equation (1) is obtained by variation of functional (9), therefore use of (1) and (9) is essentially equivalent.

As a rule, gas lenses are with long focus, and this is the reason why angles of beam inclination to the lens axis are small. This circumstance allows $\mathrm{d}s \approx \mathrm{d}z$ to be assumed with good accuracy that results in

$$\delta = -\frac{D_1}{n_0} \nabla_{r_1'} \int_{P_0}^{P_1'} n(\mathbf{r})\,\mathrm{d}z. \quad (10)$$

Consider one important case when a refractive index does not depend on z (no axial symmetry is assumed). Taking $n(\mathbf{r})$ out of the integral sign gives

$$\delta = -\frac{D_1}{n_0} \nabla_{r_1'} n'(\mathbf{r}_1')(z_1' - z_0) \quad (11)$$

[prime at $n'(\mathbf{r}_1')$ has the same meaning as primed V' in (8)]. If the incident beam front is plane, then $\mathbf{r}_1^* = 0$ (its image is a point on the axis). $D_1 = f$ if the focal length of the lens and $z_1' - z_0 = l$ is its length. Assuming $n_0 \approx 1$ and dropping out a prime at the transverse coordinate, we have for beam aberration

$$\delta = \mathbf{r}_1 = -fl\nabla n(r). \quad (12)$$

Next, using the solution obtained for temperature (7) accurate to the terms r^4 (that corresponds to an account of wave aberrations to the fourth order including), we write out the beam aberration components in an explicit form (y is the vertical axis).

$$\begin{aligned} \delta_y &= C_0 + C_2 r^2(1+\cos^2\varphi) + C_3 r^3 \cos\varphi \\ \delta_x &= d_1 r^2 \sin 2\varphi + d_2 r^3 \sin\varphi. \end{aligned} \quad (13)$$

Explicit expressions for aberration coefficients are of the form

$$\begin{aligned} C_0 &= (n_0 - 1)\frac{flq}{\lambda T} Gr\left[\frac{1}{24RePe} + \frac{1325Pr + 381}{1\,382\,400}\right] \\ C_2 &= -(n_0 - 1)\frac{flq}{\lambda T} Gr\left[\frac{1}{16RePe} + \frac{3000Pr + 735}{1\,382\,400}\right] \\ C_3 &= -(n_0 - 1)\frac{flq}{\lambda T}; \quad d_1 = C_2; \quad d_2 = C_3. \end{aligned} \quad (14)$$

The obtained expressions relate optical characteristics of thermal gas lenses to their other parameters describing flow, heating and construction peculiarities. In fact, the described lens is referred to optical systems with a symmetry plane. The axial symmetry loss, due to free convection, gives rise to a number of difficulties, say, when TGL are used as phase correctors in light guides.

In particular, aberration coefficient C_0, the so-called wedge, leads to a turn of the incident wave front, i.e. to a beam declination. Such an effect is equivalent to that of an optical wedge. It is quite natural that a similar term is absent in aberration δ_x since it is connected with natural convection effect that constantly contributes only in the vertical direction. Two other aberrations breaking spherical symmetry are characterized by coefficients $d_1 = C_2$ proportional, as well as C_1, to Grashof number and resultant from circular isotherms distortion by free convection. The character of these aberrations permits them to be identified with coma aberration, however, in contrast to the classical third-order coma all the points on the plane of an object are subjected to distortion since the second-order coma obtained here does not depend on the coordinates on the object plane. Owing to this fact, thermal gas lenses with an asymmetrical temperature field do not in general form images. Calculations of reliability of TGL light guide equivalent to calculation of the Q-factor of a resonator [1] show a considerable decrease of reliability (and Q-factor) with asymmetrical aberrations. Elimination of these aberrations is connected with suppression of free convection.

One of the effective ways of prevention from free convection is gas flow rotation. To investigate the effect of swirling a gas flow on optical characteristics of a thermal gas lens, a problem is considered of determination of a temperature field in a thermal gas lens rotating around its own axis [12]. Choose boundary conditions for system (2)–(4) in the form

$$r = 1: u = w = 0; \quad v = \frac{\omega r_0}{w_m}; \quad \left(\frac{\partial \theta}{\partial r}\right)_{r=1} = 1;$$

$$r = 0: u, w, v, \theta, \quad \text{are finite.}$$

Here ω is the angular speed of rotation of a thermal gas lens, w_m is the mean over the cross-section axial gas velocity. Assuming, as before, $\xi = Gr/Re^2$ to be a small parameter and choosing $N_\Omega = \omega r_0^2/v$ as a characteristic of swirling, we may get the velocity and temperature fields in rotating TGL.

The solution obtained [12] (to save space it is not presented) is illustrated in Fig. 1 where isotherms are presented for several values of N_Ω. It is seen that rotation does symmetrize the temperature field and, consequently, the refractive index field. Asymmetrical aberrations (wedge and coma) disappear and at a certain value of N_Ω a lens becomes practically aberration-free. The performed experimental investigations have verified the results obtained. A rotating thermal gas lens does form an image of sufficiently high quality.

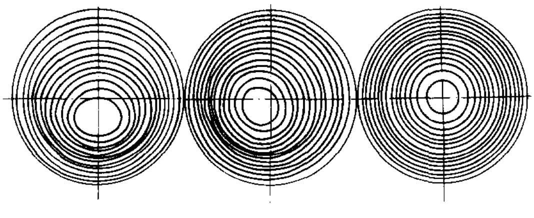

FIG. 1. Isotherms in rotating TGL; $Re = 1000$; $Gr = 500$; (a) $N = 0$; (b) $N = 10$; (c) $N = 50$.

Another way to compensate for asymmetry of optical characteristics is a non-uniform-over—perimeter heating of a lens [3], however, complete symmetrization of a temperature profile cannot be achieved in this case.

Thermal gas lenses are one of the examples of creating analogs of solid optics elements by using the aerothermooptics methods. Alongside with a design of a thermal gas lens an urgent problem is a development of analogs also for the other elements such as plane-parallel plates (chokes or aerodynamic windows), supersonic flow-based installations for light beam declination etc.

It stands to reason that the problems of aerothermooptics are not exhausted by a design of gas optics elements. The aerothermooptics problems may also comprise those involving light self-action when propagating in absorbing media [13], the problems of gravitational convection occurring with photoabsorption [14]; those of temperature behaviour of lasers since temperature fields in laser media may exert a considerable influence on the generated radiation parameters; hydrodynamics and heat-transfer problems in gasodynamic lasers.

The problem which may be called the inverse problem of aerothermooptics, namely, determination of thermophysical characteristics and heat- and mass-transfer coefficients by parameters of radiation passing through medium is also of importance. No doubt, a solution of the problems formulated by Luikov and incorporated by him into "aerothermooptics" will bring still more interesting results both in optics and in heat and mass transfer.

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L'AEROTHERMOOPTIQUE

Résumé—Une vue d'ensemble des principes et méthodes de l'aérothermooptique est présentée sur l'exemple des lentilles thermiques à gaz (TGL). La relation entre les paramètres optiques et thermohydrodynamiques des TGL est établie en ce qui concerne la convection libre. Les champs de vitesse et de température dans les TGL sont déterminés, y compris pour celles en rotation autour de leurs axes. Des expressions analytiques sont obtenues pour les coefficients d'aberration dans le développement de l'iconale jusqu'aux ondes d'aberration du quatrième ordre. La rotation de la TGL est connue pour être un moyen efficace de suppression de la convection libre et de symétrisation des propriétés optiques. Les perspectives sont discutées sur les développements ultérieurs de l'aérothermooptique.

AEROTHERMOOPTIK

Zusammenfassung—Am Beispiel der thermischen Gaslinse (TGL) wird ein Überblick über die Grundlagen und die Methoden der Aerothermooptik gegeben. Für den Fall der freien Konvektion werden die Beziehungen zwischen den optischen und den thermohydrodynamischen Parametern für die TGL aufgestellt. Dabei werden auch TGL mit Rotation um ihre Achse erfaßt. Es werden analytische Ausdrücke für die Aberrationskoeffizienten in der Eikonalentwicklung zur Erfassung von Bildfehlern bis zur 4. Ordnung aufgestellt. Die Rotation der TGL ist als wirksames Mittel zur Unterdrückung der freien Konvektion bekannt und führt zu einer besseren Symmetrie der optischen Eigenschaften. Zukünftige Entwicklungsmöglichkeiten der Aerothermooptik werden diskutiert.

АЭРОТЕРМООПТИКА

Аннотация — На примере термических газовых линз (ТГЛ) дается обзор идей и методов аэротермооптики. Устанавливается связь между оптическими и термогидродинамическими параметрами ТГЛ с учетом свободной конвекции. Найдены поля скоростей и температур в ТГЛ, в том числе вращающейся вокруг собственной оси. Получены аналитические выражения для аберрационных коэффициентов в разложении эйконала вплоть до волновых аберраций четвертого порядка. Показано, что вращение ТГЛ является эффективным средством подавления свободной конвекции и симметризации оптических характеристик. Обсуждаются перспективы дальнейшего развития аэротермооптики.